Madeira Island wakes: barotropic instability study

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Abstract

Madeira is a deep-sea island located in NE Atlantic ($33°N; 17°W$), its obstruction to the incoming oceanic and atmospheric flows induce the formation of leeward wakes. The ROMS - Regional Ocean Modeling System numerical toolkit was used, in a three-dimensional mode, to study the mesoscale ocean dynamics, leeward of Madeira Island. Results from numerical experiments showed strong vorticity generation, sensitive to the Reynolds number (Re) variability. Wake instabilities induce different behavior for cyclonic and anticyclonic eddies.

1 Introduction

Island wakes exhibit strong eddy activity with important consequences in the retention and transport of organic matter, and subsequent biological enrichment. Various physical processes such as filaments, small-scale upwelling cells or three-dimensional instabilities, enhance horizontal transport and vertical mixing of nutrient-rich deep water, when an upper surface current encounters oceanic islands [9] [10] [7] [8] [6]. The role of islands wakes in the biological enrichment of surface waters can not be neglected [11]. Motivated by this oceanographic context; by the fact that the most recent published discussion of Madeira Island geophysical wakes was [5]; and considering the recent advances in the numerical study of deep-sea island wakes [1], this talk explores the formation of oceanic island wakes, leeward of Madeira Island ($33°N; 17°W$). Recent numerical studies, [1] around an idealized cylinder suggest that an island wake in the deep ocean may not exhibit a strong cyclone-anticyclone asymmetry in barotropic instability regimes. However, the Madeira Island numerical study suggests the impossibility of asymmetric islands, such as Madeira, to generate solutions close to symmetry such as those often discussed in laboratory and idealized numerical experiments.

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2 Numerical case study

2.1 ROMS configuration

The Regional Ocean Modeling (ROMS) is a free-surface, terrain-following, primitive equations ocean model. ROMS algorithms are intensively described in [2] [3]. The hydrostatic primitive equations for momentum are solved using a split-explicit time-stepping scheme. A cosine-shape time filter, centered at the new time level is used for the averaging of the barotropic fields [3]. Time-discretized uses a third-order accurate predictor (Leap-Frog) and corrector (Adams-Molton) time-stepping algorithm. A third-order upstream biased was used for advection in order to allow for the generation of steep gradients in the solution [4].

In order to study the island wake problem in a three-dimensional mode using ROMS, a methodology similar to [1] was followed. Nevertheless, unlike [1], the schematic bathymetry was not an idealized cylinder, it represented Madeira Island (NE Atlantic). The depth was assumed uniform around the island, in order to be able to isolate the effect of the island *per se*, from the effect of the surrounding seamounts. The island was centered in a channel like configuration with a prescribed inflow at the upstream boundary such that the zonal current depended only on the vertical shear. East(E) and West(W) channel boundaries were set to slippery-tangential are zero normal conditions, whereas boundaries around the island were set to zero-normal and no-slip flow. At the southern boundary, a clamped condition was used for density and radiation conditions were used for the outgoing baroclinic flows; a numerical sponge was applied upstream of the southern open boundary; viscosity and diffusivity incremented linearly $(50 - 600m^2s^{-1})$ over the last 5% of the model domain. At the bottom, the quadratic bottom friction law was applied. During the 150 days of model calculations, bottom velocities did not influence the upper layers solutions. Initial conditions were set equal to the inflow conditions in the entire domain, except at the E-W channel walls, and at the island points. To maintain geostrophic equilibrium in interior of the domain the following assumptions were made.

The velocity profile was taken as:

$$v(z) = \frac{c_1}{2}\left[1 + \tanh\left(\frac{z + h_s}{h_d}\right)\right] \tag{1}$$

Considering the thermal wind balance, density-field is constant in y-direction and can be written as a function of x and z:

$$\rho(x, z) = \rho_0 + h(z) - \frac{\rho_0 f}{g}\int_{x_m}^{x}\frac{\partial v}{\partial z}dx \tag{2}$$

Density anomaly $h(z)$ can be taken as:

$$h(z) = \delta_\rho \tanh\left(\frac{z + h_c}{h_t}\right) \tag{3}$$

Considering the linear equation of state, neglecting salinity:

$$\rho = \rho_0 - \rho_0 c_T (T - T_0) \tag{4}$$

Temperature can be written as:

$$T(x, z) = \frac{1}{c_T} - \frac{\rho}{\rho_0 c_T} + T_0 \tag{5}$$

A linear function was used for free surface:

$$\zeta(x) = \frac{c_1 f}{g}(x - 2x_m) \tag{6}$$

Where:

Symbol	Description	Units
h_s	central depth of shear layer	m
h_d	thickness of shear layer	m
c_1	near-surface v-velocity component	ms^{-1}
h_c	central depth of thermocline	m
h_t	thickness of thermocline	m
f	Coriolis-constant	$1s^{-1}$
c_T	coefficient in linear equation of state	$^oC^{-1}$
ρ_0	mean density	kgm^{-3}
T_0	mean temperature	^oC
δ_ρ	half of (approximate) density difference between bottom and surface	kgm^{-3}
x_m	middle of domain in east-west direction	m

2.2 Re sensitivity study

The effect of an island to incoming flow is a classic problem and can be studied by considering the balance between two forces: *inertial* and *frictional*. The ratio between inertial and frictional forces is known as the Reynolds number (*Re*), a dimensional number which is written as:

$$Re = \frac{UD}{\nu} \tag{7}$$

Where U is the unperturbed upstream velocity, D the horizontal scale of the object and ν the molecular kinematic viscosity. However, geostrophy i.e. rotation and stratification, differentiates oceanic and atmospheric wakes from wakes generated in homogeneous fluids, thus eddy viscosity(ν_e, viscosity hereafter) substitutes molecular viscosity(ν) [1]; allowing for comparison between laboratory and geophysical wakes. In the Madeira Island problem, D assumes an averaged value of 40 km; U, c_1 in (1), assumes a maximum value at the surface of 0.7 ms^{-1}. In order to vary Re, ν was changed. Viscosity is calculated explicitly in ROMS therefore it can be set to zero without excessive computational noise or instability (refer to discussion in [1], section 2.2 .

For Re=100 $\nu \sim 286$; for Re=200 $\nu \sim 143$; for Re=400 $\nu \sim 72$ and for Re=800 $\nu \sim 36$. The channel was represent in a regular grid 221×351×30 ($\delta x = \delta y \sim 1.5$ km). The velocity (1) and temperatures profiles (5) assumed the following parametrization: $\rho_0 = 1027kgm^{-3}$; $T_0 = 10°C$; $c_T = 1.7^{-4}$; $g = 9.81ms^{-1}$; $h_s = 600$; $h_d = 455$; $h_t = 1300$. The model was integrated for 150 days.

Rossby number (R_0) is another dimensional parameter which is often used to evaluate the effect of planetary rotation in the length scale of a geophysical process. R_0 is defined as:

$$R_0 = \frac{U}{Df} \tag{8}$$

When R_0 is small the effects of planetary rotation are large and the net acceleration comparably small. In this case study, $R_0 \sim 0.8$, thus implying a geostrophic constrain to flow evolution. Considering the Brunt-Väisälä buoyancy frequency ($N^2 = -\frac{g}{\rho_0}\frac{\partial \rho}{\partial z} \sim 10^{-3}$) and the baroclinic deformation radius $R_d = NH/f \sim 40km$; where $H \sim 10^2$, is the vertical scale set by the upstream flow and density profiles, h_d in (1); R_d is of the same order of magnitude as D in (8), (7), (9) therefore the Burger number ($Bu \approx 1$), considering:

$$Bu = \left(\frac{R_d}{D}\right)^2 \tag{9}$$

Figure 1 (upper panel) shows an horizontal slice of surface (z=-10m) vorticity($\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$), with the flow field overlaid, for three different Re regimes. At Re $\approx$ 100, eddy shedding does not occur. Due to the asymmetric island shape vorticity is differently generated in the W side of Madeira than on the E. As Re increases the eddies develop transient meanders, detaching from the island at $Re \approx 200$. This detachment threshold is higher than that found for the symmetric island problem from [1], and certainly much higher than that found for non-rotating homogeneous flow problems i.e. $Re \approx 40$. For low Re, a weaker flow field maintains an intense re-circulation cell on the south coast. Increase Re fortifies cyclonic eddies and weakens anticyclones. As suggested by [1] the boundary shear layer around the island which is the main source of vorticity generation, is also related with changes in Re; its dependence can be fitted to a power law $Re^{-\alpha}$, where $1/4 < \alpha < 1/2$. The velocity shear near the island is the source

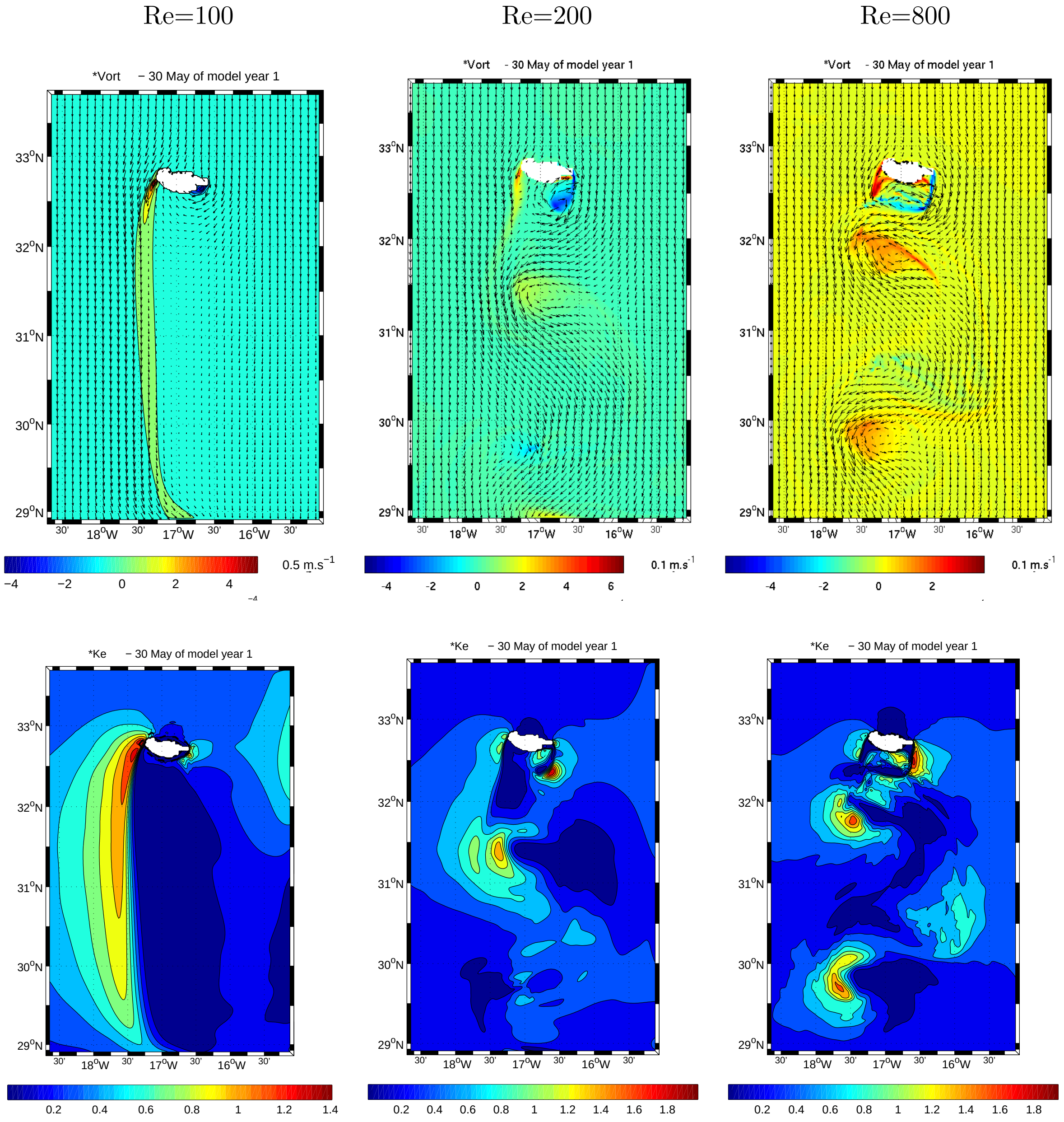

Figure 1: Horizontal slices of: (i) vorticity (upper panel) and (ii) kinetic energy (lower panel), after 150 model days for: Re=100; Re=200; Re=800

of barotropic instability such that increasing shear enhances wake instability and increases eddy kinetic energy (Ke). In fact, in figure 1 (lower panel) $Ke = 1/2(u^2 + v^2)$ varies with Re. As suggested by [1], barotropic instability implies a positive volume-integrated conversion of mean kinetic energy to eddy kinetic energy (a.k.a.'barotropic conversion'). [1], suggested that barotropic conversion is largest at one island diameter, but it can occur up to two island diameters. This is also apparent from figure 1; assuming, in this case study, that most Ke originates from eddy activity, Ke is a good indicator of EKE (Eddy Kinetic Energy). Also apparent in figure 1 are the strips of intense Ke, at the island flanks, which suggest another location of intense barotropic conversion. Whereas [1], did not found dynamical asymmetry in the product vortices, the Madeira Island case, due to the asymmetric contouring of its coastline, showed strong asymmetries between cyclonic and anticyclonic eddies.

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